QUANTUM PHYSICS I - Nov. 1, 2018

Write your name and student number on **all** sheets. There are four problems in this exam. You can earn 90 points in total.

PROBLEM 1: FOURIER TRANSFORM (5+10+10+5 points)

Consider a particle with no potential energy (i.e. a free particle) whose wavefunction at some moment is given by

$$\psi(x) = \left(\frac{2a}{\pi}\right)^{1/4} e^{-ax^2} . \tag{1}$$

a) What is the definition of the Fourier transform of an arbitrary function?

b) What is the Fourier transform of the specific wavefunction above?

c) Calculate the standard deviations σ_x and σ_p for location and momentum.

d) Indicate the time evolution of σ_x and σ_p: will these increase or decrease in time, or stay the same? You don't have to calculate anything. Explain your answer in one or two sentences.

PROBLEM 2: BACKGROUND and CONCEPTS (all 10 points)

a) Prove the relation

$$< p > = m \frac{d}{dt} < x > , \tag{2}$$

using the time-dependent Schrodinger equation.

b) Calculate

$$\frac{d}{dt} < p > , \tag{3}$$

and express this as the expectation value of a specific quantity related to the potential energy. What is the analogon of this relation in classical physics?

c) For a Hermitian operator in an infinite-dimensional vector space, are the eigenvalues always real and are the eigenvectors always normalisable? Distinguish between the cases of discrete and continuous spectra. Give an example of an operator of both cases and briefly discuss the properties of its eigenvalues and eigenvectors.

PROBLEM 3: ANGULAR MOMENTUM (all 5 points)

The spherical harmonics $Y_l^m(\theta, \phi) = \Theta(\theta)\Phi(\phi)$ with integers $|m| \leq l$ span the space of possible eigenfunctions of a two-sphere. The normalisation of eigenfunctions is not important in this problem and can be omitted.

a) What is the general form of the eigenfunctions $\Phi(\phi)$ of the operator

$$-i\frac{d}{d\phi}\,, \tag{4}$$

and what is their eigenvalue? Which boundary conditions does it satisfy?

b) The other angular dependence $\Theta(\theta)$ is governed by the operator

$$\frac{1}{\sin(\theta)}\frac{d}{d\theta}\left(\sin(\theta)\frac{d}{d\theta}\right) - \frac{m^2}{\sin^2(\theta)}\,. \tag{5}$$

What is the lowest eigenstate of this operator with $l = m = 0$ and what is its eigenvalue?

c) What is the first excited state of this operator with $l = 1$ and $m = 0$, and what is its eigenvalue? Which boundary conditions does it satisfy?

PROBLEM 4: SPIN STATES (all 5 points)

Consider the EPR proposal with pion decay of the form $\pi^0 \rightarrow e^- + e^+$. where both electron and positron move away from each other along the y-axis. The resulting spin state for the electron and positron is the entangled configuration

$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow> - |\downarrow\uparrow>) \tag{6}$$

a) What is the total spin S of this configuration?

b) Using the Pauli matrices of the formula sheet, write the one-particle z-component eigenstate $|\uparrow>$ in terms of x-component eigenstates.

c) In the Bell set-up, one measures the z-component of the electron spin, and subsequently the positron spin along a different axis. What are the probabilities of measuring the x-component of the positron spin to be up or to be down, and how are these correlated with the z-component of the electron spin?